

THE GAMMA-RAY BURST RATE AT HIGH PHOTON ENERGIES

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ABSTRACT

Some gamma-ray burst (GRB) spectra exhibit high-energy tails with the highest photon energy detected at 18 GeV. The spectral slope of the high-energy tails is sufficiently flat in νF_ν to consider the possibility of their detection at still higher energies. We calculate how many bursts can reasonably be expected above a given energy threshold for a cosmological distribution of bursts satisfying the observed apparent brightness distribution. The crucial point is that the gamma-ray absorption by pair production in the intergalactic diffuse radiation field eliminates bursts from beyond the *gamma-ray horizon* $\tau_{\gamma\gamma} \sim 1$, thus drastically reducing the number of bursts at high energies. Our results are consistent with the non-detection of bursts by current experiments in the 100 GeV to 100 TeV energy range. For the earthbound detector array MILAGRO, we predict a maximal GRB rate of ~ 10 events per year. The Whipple Observatory can detect, under favorable conditions, ~ 1 event per year. The event rate for the HEGRA array is ~ 0.01 per year. Detection of significantly higher rates of bursts would severely challenge cosmological burst scenarios.

Subject headings: diffuse radiation — gamma rays: bursts — intergalactic medium

1. INTRODUCTION

The lack of a significant large-scale anisotropy in the angular distribution of gamma-ray bursts (GRBs) argues in favor of their cosmological origin (e.g., Meegan et al. 1995; Briggs et al. 1996; Tegmark et al. 1996). Assuming they are all standard candles, the observed number of bursts at a given flux relates directly to a number of sources at a given redshift. The maximum redshift sampled by BATSE under these assumptions is $z_{\max} \sim 2$ (e.g., Cohen & Piran 1995). Evolution of the burst population can modify this value. If bursts would happen to be more active in recent cosmic history, the maximum redshift would be lower. However, the maximum redshift cannot be much lower than ~ 1 in any scenario in which the bursts trace the distribution of galaxies. Galaxy clusters are known to be concentrated toward the supergalactic plane (e.g., Tully 1991; Kolatt, Dekel, & Lahav 1995). The brightest (and thus nearest) bursts would reflect the mass concentrations of the nearby local universe. However, no significant deviation from isotropy has been found in the 3B catalog data corrected for sky exposure (Hartmann, Briggs, & Mannheim 1996).

If GRBs are indeed seen to a redshift of ~ 2 , absorption of the high-energy gamma rays by diffuse background radiation must become important at photon energies greater than 30 GeV. Since gamma-ray-emitting blazars are also subject to pair absorption, and since their redshifts are often accurately known, one can determine the density of the diffuse radiation fields from the infrared to the UV by measuring their gamma-ray cutoff energies (Stecker et al. 1992; de Jager, Stecker, & Salamon 1994; Dwek & Slavin 1994; Mannheim et al. 1996; Madau & Phinney 1996). For blazars, detailed spectral models exist which can discrimi-

nate between internal and external absorption (e.g., Blandford & Levinson 1995; Mannheim 1993).

In fact, one could calibrate the distance scale to gamma-ray bursts by measuring their cutoff energies due to external absorption using the background radiation density inferred from blazar observations. For such a program to be successful, nature must provide burst spectra reaching very high photon energies. Currently, our knowledge about high-energy burst spectra is rather sparse, and it is not known whether they exhibit an intrinsic turnover in the 10 GeV–30 TeV region or not (Hurley 1996). However, burst statistics are consistent with all BATSE bursts having high-energy tails such as the ones observed by EGRET (Dingus et al. 1995). Some of the bright GRBs which received exposure by EGRET were detected at GeV energies without evidence for a spectral rollover (Hurley et al. 1994). This raises the hope that the *intrinsic* spectra continue to still higher energies and that one could in turn use the expected exponential turnover to set a distance scale. In fact, the relativistic cosmological fireball model of Mészáros & Rees (1993) predicts that the GRB spectra have high-energy tails in general (Mészáros, Rees, & Papathanassiou 1994). Nevertheless, numerous array and Cerenkov telescope burst searches at high energies have only provided upper limits; e.g., Kieda et al. (1996) (CASA-MIA), Alexandreas et al. (1994) (CYGNUS), Aglietta et al. (1992) (EAS-TOP), Krawczynski et al. (1996), Funk et al. (1996) (HEGRA), Allen et al. (1995) (MILAGRO), Amenomori et al. (1995) (Tibet), and Connaughton et al. (1995) (Whipple).

The purpose of this work is to provide quantitative predictions of the expected burst rate as a function of photon energy accounting for cosmic absorption and to compare the result with experimental limits. We will start with the computation of the gamma-ray horizon and show templates of absorbed spectra. In a second step, we use the gamma-ray horizon to determine the number of bursts at a given detection threshold. Finally, we determine the limiting sensitivity required for a detection at a given threshold gamma-ray energy and compare this with the sensitivity of current experiments.

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2. GAMMA-RAY ABSORPTION FROM 10 GeV TO 100 TeV

Gamma rays of energy E propagating from a distant source at redshift z_0 toward a terrestrial observer can be absorbed by inelastic interactions with low-energy photons of present-day energy ϵ from an isotropic diffuse background radiation field. The dominant process is pair creation $\gamma_E + \gamma_\epsilon \rightarrow e^+ + e^-$. The threshold energy for this process is given by

$$\epsilon_{\text{th}} = \frac{2(m_e c^2)^2}{E(1 - \mu)(1 + z)^2}, \quad (1)$$

where $\mu = \cos \theta$ denotes the cosine of the scattering angle. Since the soft photon density varies strongly with energy for typical radiation fields, the optical depth $\tau_{\gamma\gamma}$ must also vary reflecting the number density of target photons at the resonant energy $\propto E^{-1}$. The pair creation cross section is given by

$$\sigma_{\gamma\gamma} = \frac{3\sigma_T}{16} (1 - \beta^2) \left[2\beta(\beta^2 - 2) + (3 - \beta^4) \ln \left(\frac{1 + \beta}{1 - \beta} \right) \right] \text{cm}^2, \quad (2)$$

where $\beta = (1 - 1/\gamma^2)^{1/2}$ with $\gamma^2 = \epsilon/\epsilon_{\text{th}}$, and where σ_T denotes the Thomson cross section. For the computation of the optical depth, we use the geodesic radial displacement function $dl/dz = c/H_0[(1+z)E(z)]^{-1}$, where $E(z)$ is given by equation (13.3) in Peebles (1993). For a cosmological model with $\Omega = 1$ and $\Lambda = 0$, the function $E(z)$ simplifies to $(1+z)^{3/2}$. Hence, we obtain the optical depth

$$\begin{aligned} \tau_{\gamma\gamma}(E, z_0) &= \int_0^{z_0} dz \frac{dl}{dz} \int_{-1}^{+1} d\mu \frac{1 - \mu}{2} \int_{\epsilon_{\text{th}}}^{\infty} d\epsilon n_b(\epsilon) \\ &\quad \times (1 + z)^3 \sigma_{\gamma\gamma}(E, \epsilon, \mu, z) \\ &= \frac{c}{H_0} \int_0^{z_0} dz (1 + z)^{1/2} \int_0^2 dx \frac{x}{2} \int_{\epsilon_{\text{th}}}^{\infty} d\epsilon n_b(\epsilon) \sigma_{\gamma\gamma} \\ &\quad \times (E, \epsilon, x - 1, z) \end{aligned} \quad (3)$$

for a nonevolving present-day background density n_b , i.e., $n'_b(z, \epsilon) d\epsilon' = (1 + z)^3 n_b(\epsilon) d\epsilon$, where the dash indicates comoving-frame quantities. At some time in the past, the background photon density was built up by bursts of massive star formation during the era of galaxy formation. As a result, the photon density evolution should be more shallow than $\propto (1 + z)^3$ at this epoch, and the corresponding γ -ray absorption should be somewhat weaker. We will discuss this effect only qualitatively below. The shape of the present-day diffuse background density $n_b(\epsilon)$ is obtained by averaging over various galaxy formation models presented by MacMinn & Primack (1996). We multiplied this shape by a small factor to obtain agreement with the background density (including the contribution from the 2.7 K microwave background) estimated by Beichman & Helou (1991) in the far-infrared (FIR) (for a modest galaxy luminosity or density evolution $\gamma = 2$), and by Madau & Phinney (1996) in the near-infrared (NIR) through UV range (Fig. 1). The predicted background densities depend sensitively on which density or luminosity evolution and maximum redshift are assumed and can vary by an order of magnitude. Hence, it is very important to tighten existing constraints on the actual background density (Biller et al. 1995) by obtaining

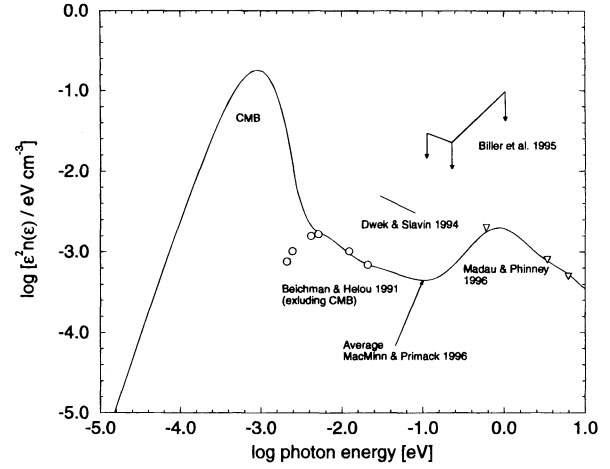


FIG. 1.—Spectral energy distribution of the diffuse background radiation. Components are the 2.7 K cosmic microwave background (CMB), dust, and stellar light from galaxies. *Solid line*: FIR-to-UV background adopted in the present work (which closely resembles the spectrum obtained by averaging over the various cold + hot dark matter models of galaxy formation of MacMinn & Primack 1996). *Straight line segment*: FIR background photon density inferred by Dwek & Slavin (1994) assuming TeV absorption for Mrk 421. *Limits*: Experimental upper limits on the background IR density obtained by Biller et al. (1995).

better galaxy luminosity functions from deep galaxy surveys or by direct measurements of blazar gamma-ray (external!) turnover detections.

We integrate numerically the optical depth function and solve for the gamma-ray horizon $\tau_{\gamma\gamma}(E_0, z_0) = 1$. Results are shown in Figure 2 for two values of the Hubble constant. Templates of absorbed infinite power-law spectra are shown in Figure 3 for various redshifts.

Note that the adopted diffuse radiation background does not have a strong dust component which would be responsible for TeV absorption of Mrk 421, as suggested by de Jager et al. (1994). Theoretical modeling of the broadband continuum spectrum of Mrk 421 over 18 orders of magnitude in frequency predicts a cutoff due to *internal* gamma-ray absorption for Mrk 421 (Mannheim et al. 1996) which could explain the lack of gamma rays above TeV observed by Whipple. This is in contrast to the assumption by de Jager et al. (1994) of a strict power law for the

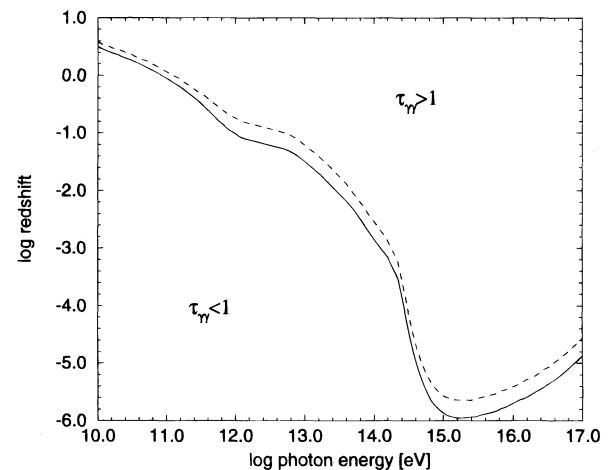


FIG. 2.—Gamma-ray horizon $\tau_{\gamma\gamma} = 1$ for the diffuse background radiation shown in Fig. 1. *Solid line*: $h = 0.5$. *Dashed line*: $h = 1.0$.

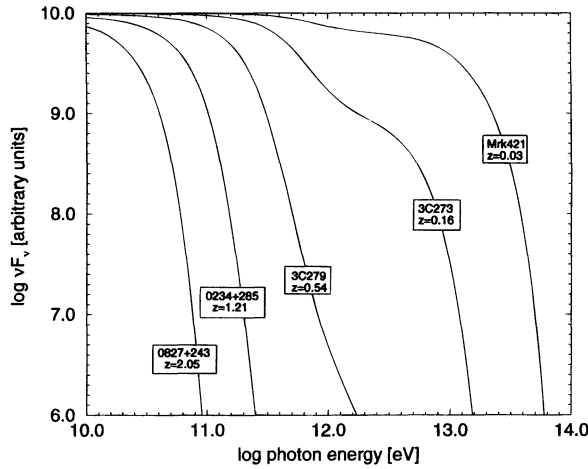


FIG. 3.—Templates of absorbed $\alpha = 2$ spectra at various redshifts at which blazars have been detected with EGRET ($h = 0.5$).

intrinsic spectrum extending well beyond TeV. Further observations will show whether the cutoff is variable with flux as expected from the intrinsic model or whether it is fixed as expected from absorption by an anomalously high external dust component.

3. EXPECTED TOTAL NUMBER OF BURSTS

In our straw person's model, the bursts represent standard candles out to a redshift of $z_{\max} = 2.1$ (Cohen & Piran 1995). Assuming no source evolution and a galaxy density of $\sim 10^{-2} h^3 \text{ Mpc}^{-3}$ ($H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$), the observed BATSE brightness distribution can be matched if $\sim 10^{-6}$ bursts (each of energy $L_0 = 7 \times 10^{50} h^{-2} \text{ ergs}$) occur per year per galaxy. The number of bursts above a gamma-ray energy E_0 is then given by the number of bursts in the cosmic volume enclosed by the gamma-ray horizon at that energy, i.e., the hyperplane $\tau_{\gamma\gamma}(z_0, E_0) = 1$. The absorbed energy from bursts beyond the horizon is re-emitted in other wave bands depending on the intergalactic magnetic field (Aharonian, Coppi, & Völk 1994), possibly with time delays indicative of intergalactic magnetic fields (Plaga 1995). For burst spectra as steep as those observed with EGRET in the GeV range (Hurley et al. 1994), the “pileup” of absorbed energy below E_0 is negligible. The

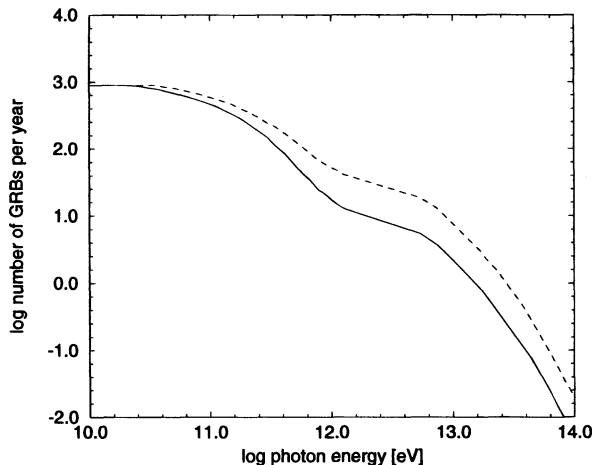


FIG. 4.—GRB rate as a function of energy threshold. Solid line: $h = 0.5$. Dashed line: $h = 1.0$.

number of bursts per year at a given threshold energy E_0 and for a volume burst rate n_0 is then given by

$$N(\geq E_0) = 4\pi n_0 \left(\frac{c}{H_0} \right)^3 \int_0^{z_0} dz \frac{y(z)^2}{(1+z)E(z)} \quad (4)$$

[see eq. (13.61) of Peebles (1993), with an extra factor $(1+z)$ accounting for the redshift of the burst rate]. The integral simplifies considerably for $\Omega = 1$ and $\Lambda = 0$. In this case, the angular size distance $y(z)$ is given by $y(z) = 2[1 - (1+z)^{-1/2}]$ and $E(z) = (1+z)^{3/2}$, yielding

$$N(\geq E_0) = 16\pi n_0 \left(\frac{c}{H_0} \right)^3 \left[\frac{1}{6} + \frac{1}{2(1+z_0)^2} - \frac{2}{3(1+z_0)^{3/2}} \right]. \quad (5)$$

For the volume burst rate $n_0 = 6.9 \times 10^{-9} h^3 \text{ Mpc}^{-3} \text{ yr}^{-1}$ and a BATSE trigger efficiency of $N_{\text{BATSE}}/N = 0.3$, we obtain $N_{\text{BATSE}} = 300 \text{ yr}^{-1}$ integrating to $z_0 = z_{\max}$. The dependence on the Hubble constant enters equation (4) only through the value of $z_0 = z_0(E_0, H_0)$ and is therefore rather weak. Nevertheless, it is an interesting possibility to determine the Hubble constant by number counts of GRBs.

In Figure 4 we show the number of bursts versus detection threshold demonstrating the rapid shrinking of the observable universe with increasing E_0 due to pair absorption. At energies of $\sim 10 \text{ TeV}$, the standard-candle scenario predicts 2–4 ($h = 0.5$ – 0.75) bursts per year.

4. LIMITING SENSITIVITY

The number of bursts which can be detected above a given energy threshold depends also on the particular instrumental sensitivity F_0 , the spectral shape, and the duration of the bursts. Suppose we want to measure all GRBs above an energy E_0 (within redshifts $z < z_0$); then what is the necessary flux sensitivity of our detector to achieve this goal? In the straw person's model, this problem is easy to solve. For a given threshold E_0 , the observed flux obeys the relation

$$F_0 = \frac{L[\geq E_0(1+z_0)]}{4\pi(c/H_0)^2 y(z_0)^2 (1+z_0)^2}, \quad (6)$$

again referring to Peebles's (1993) notation. The standard-candle luminosity normalized for a BATSE burst duration δt is given by

$$L[\geq E_0(1+z_0)] = 7 \times 10^{50} h^{-2} \left(\frac{\delta t}{1 \text{ s}} \right)^{-1} \times \left(\frac{E_0}{E_b} \right)^{2-\alpha} (1+z_0)^{2-\alpha}, \quad (7)$$

adopting a high-energy power-law spectrum with photon index $\alpha > 2$ above an intrinsic break energy $E_b \sim 1 \text{ MeV}$. With $\Omega = 1$ and $\Lambda = 0$, this corresponds to

$$F_0 = 1.7 \times 10^{-7} \frac{(1+z_0)^{1-\alpha}}{(\sqrt{1+z_0}-1)^2} \left(\frac{\delta t}{1 \text{ s}} \right)^{-1} \left(\frac{E_0}{E_b} \right)^{2-\alpha} \text{ ergs cm}^{-2} \text{ s}^{-1}, \quad (8)$$

for the source flux at the distance of the gamma ray horizon. All other sources will be within the horizon, thus closer, and thus brighter. Although the burst luminosity in the standard-candle picture is fixed ($\delta t \sim 1 \text{ s}$), we leave the

possibility open that the high-energy tails persist longer than the typical BATSE bursts. Hurley et al. (1994) indicate spectral slopes $\alpha = 2.2$ –3.7 and extended or delayed durations of 30 s–90 minutes for EGRET-detected bursts.

In Figure 5 we plot the limiting sensitivity for $\alpha = 2.2$ and $\alpha = 2.7$ and for high-energy tail durations extended relative to the BATSE bursts by factors 1 and 100. For comparison, we show current experimental limits in the same plot demonstrating that they can in principle detect all bursts. The apparently paradoxical increase of the limiting flux with energy reflects the rapid shrinking of the gamma-ray horizon: the higher the energy, the closer, and thus the brighter, the bursts from the gamma-ray horizon are. More pessimistically, if the intrinsic spectra were steeper than the EGRET bursts, the sensitivity of current experiments is insufficient to detect all of them. In view of the very small *total* burst rates at high energies (see Fig. 4), this would practically comply with zero expected event rates.

5. CONCLUSIONS

We estimated the expected gamma-ray burst rate as a function of threshold energy. The straw person's scenario considers standard candles at cosmological distances emitting an intrinsically unabsorbed high-energy power-law spectrum. With these simplifying assumptions, we predict a maximum of 20–40 bursts per year in the TeV range, roughly an order of magnitude less above 10 TeV, and practically none at 100 TeV. The perhaps more realistic case of a steep power-law luminosity distribution of the bursts does not significantly affect the maximum redshift z_{max} of their cosmological distribution (Cohen & Piran 1995), and therefore it does not change the expected burst rates. An enhancement of the burst rate in the 10–100 GeV range by factors of ~ 2 is possible if evolution of the background density is important at redshifts ~ 1 . The sensitivities of current experiments are sufficient to detect bursts if their spectra are not steeper than $\alpha \sim 2.7$. Taking into account sky exposure and triggering efficiency (Table 1), we rule out the possibility of cosmological burst detection by CYGNUS, EAS-TOP, and CASA-MIA (and similar experiments with thresholds at ~ 100 TeV). Due to its low threshold, MILAGRO or future low-threshold air Cerenkov telescopes could at most detect ~ 10 events per year. The

TABLE 1

PREDICTED GAMMA-RAY BURST RATES FOR VARIOUS EXPERIMENTS

Experiment	Trigger Efficiency	Energy Threshold (TeV)	GRB Rate (yr ⁻¹)
CASA-MIA	0.05	100	0.0003
CYGNUS	0.05	100	0.0003
EAS-TOP	0.05	100	0.0003
HEGRA-Scintillators	0.05	25	0.02
CASA-DICE	0.0004	20	0.0002
HEGRA-AIROBICC	0.006	13	0.01
MILAGRO	0.05	0.2	10
Tibet	0.05	10	0.1
Whipple ^a	0.006	0.3	1.0

NOTE.— $\Omega = 1$, $\Lambda = 0$, $h = 0.5$. A Hubble constant $h = 0.75$ would increase all rates by a factor ~ 2 .

^a In order to start follow-up observations, slewing the Cerenkov telescope can take up to 1 hr.

Whipple Observatory, the Tibet air shower array, and HEGRA have very low expected detection rates (0.01–1 events per year). If the burst delays at high energies are shorter than the telescope slewing time, the expected burst rate for air Cerenkov telescopes would be down by ~ 0.01 owing to their small field of view. The major uncertainties of the theoretical expectation values lie in the cosmic IR-to-UV photon density (~ 10 in the FIR), the Hubble constant (factor < 2), and the effect of forward cascading in the intergalactic medium which counteracts the pair absorption depending on the intergalactic magnetic field strength (Protheroe & Stanlev 1993; Aharonian et al. 1994; Plaga 1995). A low background density, a large Hubble constant, and weak intergalactic magnetic fields would act to increase the high-energy burst detection rate. On the other hand, the high IR background inferred by de Jager et al. (1994) and Dwek & Slavin (1994) assuming TeV absorption of Mrk 421 would reduce further the number of bursts above ~ 1 TeV by a factor of ~ 10 . In any cosmological scenario, pair absorption by diffuse background radiation leads to a rapidly decreasing number of bursts above ~ 10 GeV. By contrast, bursts originating in a galactic halo would not be affected by absorption which occurs only on cosmic scales. Halo bursts should therefore be much more numerous than cosmological bursts at high energies, as has been pointed

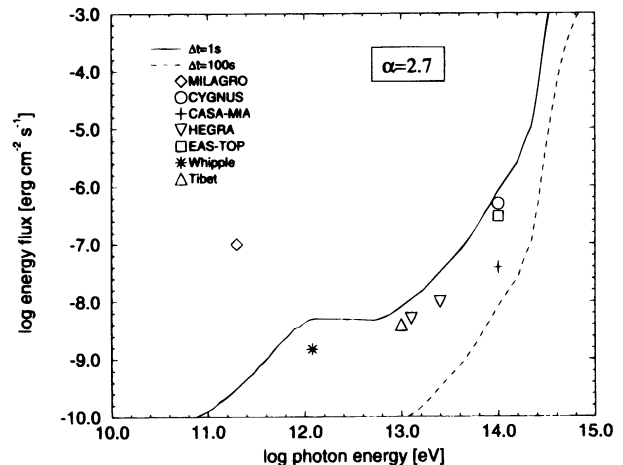
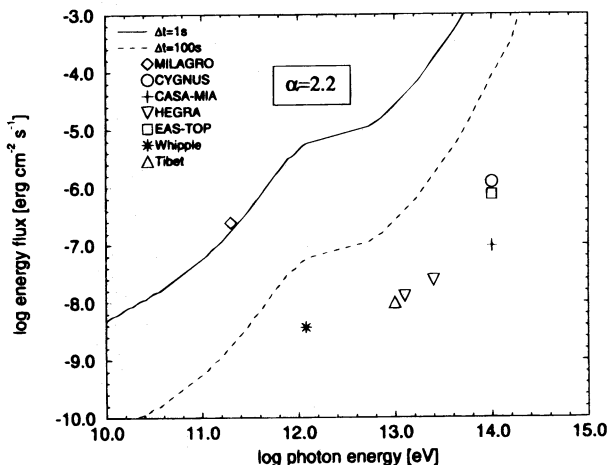


FIG. 5.—Solid lines: Limiting sensitivity for detection of all GRBs as a function of threshold energy assuming a high-energy tail spectral index α ($h = 0.5$). Dashed lines: Same for high-energy tails with extended durations relative to the BATSE bursts by a factor of 100. Symbols: current experimental flux limits for burst detection (typically assuming $\delta t \sim 30$ s).

out by Alexandreas et al. (1994) and Hurley (1996). In fact, if one clings to the assumption of infinite power law tails, the nondetection of GRBs by current experiments is in agreement with the straw person's scenario of a cosmological origin of bursts, but it would require intrinsic spectral cutoffs below ~ 100 GeV in halo models. On the other hand, just a few burst detections above ~ 50 TeV would render cosmological models highly unlikely. If intrinsic cutoffs would be important, it is interesting to note that the predictions for halo and cosmological burst models are again quite different. In halo models, the brighter bursts should have cutoffs at lower energies due to their higher compactness, whereas cosmological models predict the opposite, since the bright sources would be near sources

with less external absorption. This crucial test could be performed by future missions that target the favorable energy range 10–100 GeV.

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